

Turbulence-induced magnetic fields in shock precursors

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ABSTRACT

Galactic cosmic rays are believed to be mostly accelerated at supernova shocks. However, the interstellar magnetic field is too weak to efficiently accelerate galactic cosmic rays up to the highest energies, i.e. 10^{15} eV. A stronger magnetic field in the pre-shock region could provide the efficiency required. Bell’s cosmic-ray nonresonant streaming instability has been claimed to be responsible for the amplification of precursor magnetic fields. However, an alternative mechanism has been proposed in which the cosmic-ray pressure gradient forms the shock precursor and drives turbulence, amplifying the magnetic field via the small-scale dynamo. A key ingredient for the mechanism to operate are the inhomogeneities present in the interstellar medium (ISM). These inhomogeneities are the consequence of turbulence. In this work we explore the magnetic field amplification in different ISM conditions through 3D MHD numerical simulations.

Key words: Acceleration of particles — cosmic rays — MHD — shock waves — turbulence — methods: numerical

Cosmic rays (CRs) are high-energy particles with energies between $10^8 - 10^{22}$ eV whose origin has been subject of debate since the beginning of research in the field (e.g., Ginzburg & Syrovatskii 1964). From energetic constraints it can be shown that CRs with energies up to 10^{15} eV (the so called *knee*) are of galactic origin (see Blasi 2013, and references there in). The main galactic CR sources are believed to be supernova remnants (SNRs) shocks, where particles are accelerated through the diffusive shock mechanism (e.g., Bell 1978; Drury 1984).

The pressure of CRs can be comparable to the pressure of the background medium, so they produce dynamical effects on the background plasma and magnetic field (e.g., Axford, Leer & McKezie 1982); CRs modify shocks (see, Malkov & Drury 2001; Blasi 2013, and references there in). The accelerated particles can extract significant energy from the shocked flow. The high-energy particles diffuse ahead of the shock producing a pressure gradient upstream of the shock transition that smoothly decelerates and compresses flow into the shock, and a shock precursor is formed.

A serious problem on the theory of CR acceleration in SNR shocks is that the magnetic field strengths in the upstream region, i.e. the $\sim 5 \mu\text{G}$ ISM magnetic fields,

are too weak for efficiently accelerate CRs of PeV energies (Lagage & Cesarsky 1983). PeV CRs have long mean free paths in such fields and have a high probability of escaping the shock, so they are not subject to further acceleration. The problem might be solved if the magnetic field in the pre-shock region can be much stronger than its interstellar value and if the free energy available for the shock is sufficient to generate much larger fields.

Furthermore, indirect observational arguments require higher magnetic fields at the SNR shocks, higher than expected by simple shock compression. Detection of rapid variability in X-ray emission from a supernova remnant indicates an amplification factor of the magnetic field of more than 100 (Uchiyama et al. 2007). Upstream magnetic field amplification not only should occur in SNR shocks, but in other strong shock waves in the ISM produced, for example, by massive star winds, stellar outflows, jets, etc. Also observational evidence suggests that radio relics host relatively large magnetic fields (Markevitch et al. 2005; van Weeren et al. 2011). Magnetic field amplification might occur at the weak shocks that produce the radio relics (Brüggen 2013).

Bell (2004) proposed the current driven instability to overcome this problem. The driving electric current of the instability comes from drift (streaming) of the escaping CRs. The reacting current of the background plasma leads to a transverse force that can amplify transverse perturbations in the magnetic field. However, this mechanism can only generate fields on scales smaller than the gyro-radius

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of the driving particles, and the generated fields are on scales too small to be used to accelerate the highest energy particles (Schure et al. 2012; Beresnyak & Li 2014). Other process might occur simultaneously. Another instability has been proposed (e.g., Diamond & Malkov 2007; Malkov & Diamond 2009), the Drury acoustic instability (Drury 1984), which is the enhancement of compressible perturbations by the CR pressure gradient. This instability potentially can have faster growth rates than the Bell instability and operates on larger scales. The relative role of different instabilities in generating magnetic field in front of the shock is currently the field of active research.

Beresnyak, Jones & Lazarian (2009) (henceforth BJL09) have presented an alternative process that can provide fast magnetic field generation. In the BJL09 model the magnetic field is generated by purely non-linear fluid mechanisms. The CR pressure gradient accelerates differentially the denser and lighter parts of the ISM upstream flow carrying density inhomogeneities, generating in this way local shear or vorticity. This vorticity drives a (supersonic) turbulence cascade. The magnetic energy is amplified on scales of the order and below the scales of the vorticity structures (of the order of the density structures scales) through the small-scale dynamo (e.g., Cho et al. 2009; Beresnyak & Li 2014). This idea was later elaborated by Drury & Downes (2012).

The inhomogeneities observed in the ISM covering an extended range of scales, the spectrum of these density fluctuations are consistent with Kolmogorov turbulence, the so-called Big Power Law in the Sky (Armstrong, Rickett & Spangler 1995; Chepurnov & Lazarian 2010); density inhomogeneities are present in many astrophysical plasmas and are usually associated with magnetohydrodynamic (MHD) turbulence (e.g., Elmegreen & Scalo 2004; Lazarian & Opher 2009). The ISM is composed by different phases according to the physical conditions of the material. These physical conditions establish different regimes of MHD turbulence. The typical values for the density fluctuations amplitudes and scales from the turbulent ISM would allow a magnetic field amplification larger in scales and strength than that provided by the Bell's instability (Beresnyak, Jones, & Lazarian 2009).

Drury & Downes (2012) presented the first study on the turbulent magnetic field amplification driven by CR pressure gradients using 2D MHD simulations; later on they extended this work and compared the results with 3D simulations (Downes & Drury 2014). They found that independent of the detailed plasma physics, as long as there is a cosmic ray precursor with a significant associated pressure gradient, and assuming the inflowing medium is inhomogeneous, the magnetic field can be amplified in the precursor by a factor $\sim 5 - 20$ (depending mainly on the direction of the uniform initial magnetic field).

Brüggen (2013) also studied upstream magnetic field amplification by turbulence using 3D and mostly 2D MHD simulations; weak shocks of galaxy clusters were also considered. However, due to the amplification relying on the hydromagnetic turbulence the 2D treatment is not adequate (because the physics of the turbulent cascade is different from the 3D real case). In addition, the above mentioned

works considered a synthetically built density fluctuations spectrum for representing the medium inhomogeneities.

In this work we also study turbulence-induced magnetic field amplification in shock upstream, exploring the details of the turbulence developed in the precursor. We use density structures obtained in different regimes of compressible MHD isothermal turbulence simulations to represent the inhomogeneities in the medium swept by the shock, in order to compare with the simplest density structures built artificially. We aim at studying the magnetic field amplification for different conditions of the ISM, under the ideas presented by BJL09. Here we deal only with magnetic field amplification by turbulence in the pre-shock region. Post-shock turbulence can further amplify the magnetic field downstream; this situation is studied elsewhere (e.g., Giacalone & Jokipii 2007; Gou et al. 2012; Mizuno et al. 2014).

This paper is organized as follows. In the next section we describe the magnetic field amplification model developed in BJL09 and we highlight the existence of the density homogeneities in the ISM, one of the main ingredients of the mechanism. Section 2 describes the numerical setup we adopt in the MHD simulations. In the Section 3 we present the results and data analysis. An extended discussion is made in Section 4. Finally, in Section 5, we draw our conclusions.

1 MAGNETIC FIELD AMPLIFICATION MECHANISM

Here we describe briefly the main ideas in the model developed in BJL09 for magnetic field amplification. For further details and argumentations the reader is referred to the original paper.

The fluid elements passing through the precursor have density inhomogeneities. As already mentioned, some level of density inhomogeneity in astrophysical fluids is naturally generated by the combination of compressible turbulence, gravity and cooling (Armstrong, Rickett & Spangler 1995; Burkhart et al. 2009; Federrath et al. 2010; Chepurnov & Lazarian 2010). When this inhomogeneous plasma flows into the precursor with speed v_0 (the shock front velocity in the reference frame of the upstream flow), it is decelerated by the CR pressure gradient down to a velocity v_1 at the shock front. This situation is illustrated in Fig. 1. The interaction between the fluid and the CR pressure creates perturbations of velocity $\sim (v_0 - v_1)$, as a consequence of the differences between the ballistic velocity that highest density regions have and the full deceleration suffered by the lowest density regions. The resulting velocity field is partially solenoidal due to the shear caused by the different velocities acquired by the light and dense parts of the gas.

The amount of solenoidal velocity can be parametrized as

$$v_s = A_s (v_0 - v_1), \quad (1)$$

with $A_s \leq 1$. The characteristic scale of the solenoidal perturbations l is, for the case of supersonic ISM, the distance between slow sub-shocks (see Section 1.1), which is smaller than the turbulence outer scale l_0 . l can be of the order of parsecs (see discussion in the next subsection). These

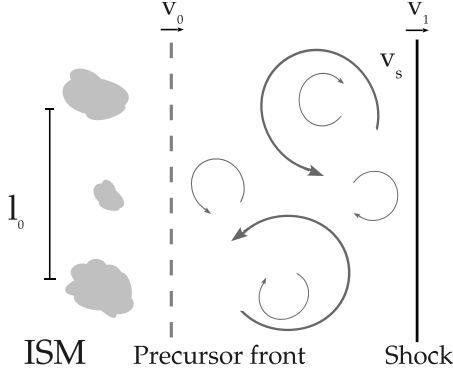


Figure 1. 2D scheme of the solenoidal motions excited by cosmic-ray pressure in the shock precursor.

solenoidal velocities develop a turbulent cascade, which amplify the weak magnetic field embedded in the precursor through the turbulent Small-Scale Dynamo (SSD).

The SSD has three main stages: the kinematic stage during which magnetic energy grows exponentially; the linear stage and a saturation stage (e.g., Cho et al. 2009; Beresnyak & Li 2014). The kinematic stage in astrophysical problems is not relevant because its saturation time-scale is very short compared to astrophysical time-scales. Therefore, it is always assumed that the kinematic dynamo is saturated and the dynamo is in the linear stage. In this stage the magnetic energy increases linearly with time, i.e.,

$$\frac{1}{8\pi} \frac{dB^2}{dt} = A_d \epsilon, \quad (2)$$

here ϵ is the energy transfer rate of the turbulence and A_d can be thought as the efficiency of the SSD. The energy transfer rate can be estimated as $\epsilon = \rho v_s^3/l$.

At some time the magnetic field reaches equipartition with the turbulent velocity field for motions at the scale l^* . This scale grows with time. On scales smaller than l^* magnetic and velocity perturbations form a MHD turbulent cascade with a steep spectrum. On scales larger than l^* , the magnetic field has a shallow spectrum and the velocity has a Kolmogorov spectrum.

When l^* approaches l , the SSD enters the saturation stage in which the magnetic field grows more slowly than in the previous linear stage. However, in the physical scenario considered here, the saturation stage is never reached, since limited time is available for field amplification.

The mechanism efficiency depends on the relative amplitude of the initial density perturbations, i.e. $\delta\rho/\rho$. When $\delta\rho/\rho \sim 1$, the mechanism is expected to be more efficient. Even when the ratio $\delta\rho/\rho$ is initially small, the initial density perturbations could be enhanced in the precursor up to the order of unity by different mechanisms. Under some conditions Rayleigh-Taylor-like instabilities can strongly enhance turbulence in the CR precursor.

1.1 Turbulence in the interstellar medium

Many possible sources of turbulence in the ISM exist operating at different scales and injecting different powers. The ISM is classified in phases, according to the temperature and state of the Hydrogen, of which it is almost fully composed.

Cold regions in which hydrogen is in molecular form include dark clouds and molecular clouds, and in which hydrogen is atomic include cold neutral medium, photo-dissociation regions, reflection nebula, etc. The warm ISM includes warm neutral medium of atomic hydrogen and ionized regions such as the so-called warm ionized medium, and H II regions. The hot ISM includes coronal gas, hot ionized medium, etc. (e.g., Draine & Lazarian 1998). The physical conditions of the ISM phases, including the degree of magnetization, establish the sonic and Alfvénic regime of the turbulence.

The turbulence in hot and warm ISM is usually subsonic or trans-sonic with respect to the ambient turbulent motions (see Hill et al. 2008; Burkhart et al. 2012). A generic property of the cold ISM is cooling which keeps temperatures low, making ISM turbulence highly compressive. Thus the Mach numbers increase in cold HI (see Chepurnov et al. 2010). In dense regions, observations reveal that turbulence in molecular clouds and star forming regions can be highly super-sonic (see Crutcher et al. 1999; Falgarone et al. 2008). In general, the warm ISM typical observed Mach numbers are between 1 and 10. The cold and dense ISM has Mach numbers between 10 and 50 (e.g., Mac Low & Klessen 2004).

2 NUMERICAL SETUP

In order to study the magnetic field amplification on the shock precursor we perform MHD simulations of a super-sonic flow in the shock referential frame. We employ the PLUTO code (Mignone et al. 2007), a conservative shock-capturing code, for solving the dimensionless ideal MHD equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (3)$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v} + P) + \mathbf{B} \times (\nabla \times \mathbf{B}) = \mathbf{F}_{\text{CR}}, \quad (4)$$

$$\frac{\partial e}{\partial t} + \nabla \cdot \left[\left(e + P + \frac{B^2}{2} \right) \mathbf{v} - \mathbf{B}(\mathbf{v} \cdot \mathbf{B}) \right] = \mathbf{F}_{\text{CR}} \cdot \mathbf{v} \quad (5)$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0, \quad (6)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (7)$$

where ρ is the density, \mathbf{v} is the velocity, P is the thermal pressure, \mathbf{B} is the magnetic field, $e = P/(\gamma - 1) + \rho v^2/2 + B^2/2$ the total energy density (excluding the CR energy), γ is the adiabatic index, and \mathbf{F}_{CR} is the force per volume due to the CR pressure gradient (see below). We use the third order Runge-Kutta scheme for time evolution, and the fluxes are calculated employing the HLL solver and parabolic interpolation. The magnetic divergence is controlled using a cleaning divergence scheme. The MHD equations are solved in a three-dimensional Cartesian box discretized by a uniform grid.

The CR pressure is directly related to the flow kinetic energy available in the pre-shock (at the shock referential). In our simulations the shock front is considered flat and localized at the right boundary of the computational domain. We model the force due to the CR pressure \mathbf{F}_{CR} as de-

ing exponentially with the distance from the shock position x_s ¹, through the equation:

$$\mathbf{F}_{\text{CR}} = -\frac{\eta}{D_{\text{CR}}} \langle p_x(x_s) \rangle \langle v_x(x_s) \rangle e^{-|x-x_s|/D_{\text{CR}}} \mathbf{i}. \quad (8)$$

Here $\langle p_x(x_s) \rangle_{yz} = \langle \rho(x_s) v_x(x_s) \rangle_{yz}$ and $\langle v_x(x_s) \rangle_{yz}$ are the averages in the yz -plane of the x -component of the momentum and of the x -component of the velocity respectively; η is an efficiency parameter smaller than 1. Following Drury & Downes (2012) (see also Vink 2012), we adopt $\eta = 0.6$. The CR diffusion scale $D_{\text{CR}} = \kappa_{\text{CR}}/v_0$, where κ_{CR} is the CR diffusion coefficient in the preshock region and v_0 is the preshock flow velocity, is considered constant in the entire domain and in time. The computational domain, in the direction parallel to the shock propagation (x -direction), has length $L = D_{\text{CR}}$.

We apply the physical constraint that the force produced by the CR pressure is halted if the total Mach number² $\mathcal{M}_T = \langle v_x(x_s) \rangle_{yz} / \sqrt{\langle c_s(x_s) \rangle_{yz}^2 + \langle v_A(x_s) \rangle_{yz}^2}$, where $v_A = B/\sqrt{4\pi\rho}$ is the local Alfvén velocity, becomes less than unity close to the shock (although this condition is never fulfilled in our simulations).

It should be remarked that we do not use explicit viscosity and/or resistivity in our calculations in order to maximize the Reynolds and magnetic Reynolds numbers, which are very high in astrophysical flows³. We do not include heat conduction and/or cooling in our simulations.

2.1 Initial conditions

The domain is initially filled by a density field ρ composed by a uniform component plus an inhomogeneous, “turbulent” one: $\rho = \rho_0 + \delta\rho_{\text{turb}}$, with $\langle \rho \rangle = \rho_0 = 1$ (in code units), where the brackets $\langle \cdot \rangle$ denote a spatial average over the domain. The initial pressure is uniform and given by $P = c_s^2 \rho_0 / \gamma$, where $c_s = 1$ (in code units) is approximately the initial average sound speed, and we use the adiabatic index $\gamma = 5/3$ in all our simulations. The initial flow has a uniform velocity $v_0 = 100c_s$ in the x -direction. The initial magnetic field is uniform in the y direction, i.e. a perpendicular shock, with $B_0 = 0.1$.

We define the initial sonic Mach number and Alfvénic Mach number in the preshock respectively as $\mathcal{M}_s \equiv v_0/c_s$ and $\mathcal{M}_A \equiv v_0/B_0$. Therefore, through this study we use $\mathcal{M}_s = 100$ and $\mathcal{M}_A = 1000$, unless stated otherwise.

We parametrize the “turbulent” density fluctuations $\delta\rho_{\text{turb}}$ used in the initial condition by two dimensionless quantities: the ratio between the typical length of the density

structures and the CR diffusion scale $\lambda_\rho/D_{\text{CR}}$, and the relative amplitude of these density fluctuations $\delta\rho_{\text{rms}}/\rho_0$ (the density contrast).

Table 1 shows the parameters of the models presented in this work. The initial density fluctuation fields are calculated in different ways: for Models A, a spectrum of plane waves with random phases is imposed (synthetic structures), and for Models B, the density field is taken from a snapshot of MHD turbulence simulations. These two different initial conditions are detailed below.

2.1.1 Synthetic density fluctuations

Using a density field built from a composition of random-phased wave modes with an imposed spectrum allows us to directly control the key parameters λ_ρ and $\delta\rho_{\text{rms}}$ of the density structures. According to the BJL09 theory, both quantities should impact on the magnetic energy growth due to different reasons: the highest the density contrast $\delta\rho_{\text{rms}}/\rho_0$ the more intense vorticity the CR pressure generates in the flow, and the smaller the density structures size λ_ρ , the smaller the turn-over times of the largest eddies of the induced turbulence, which means that the turbulence cascade has more time to develop and to amplify the magnetic field before the plasma crosses the shock.

The initial density field is a log-normal distribution given by

$$\rho(x, y, z) \propto \rho_0 \exp(C\delta f(x, y, z)) \quad (9)$$

where $\delta f(x, y, z)$ is generated by summing over a larger number of discrete modes with the wave-vectors norm k between $k_{\text{min}} = L/\lambda_\rho$ (the wave-number characterizing the density fluctuations scale) and $k_{\text{max}} = 2L_y/N_y$ (the largest wave-number in the box). These modes have random phases and their amplitudes are calculated so that the final unidimensional power spectrum of δf follows the power law $P(k) \propto k^{-5/3}$. The amplitudes of the modes with k smaller than k_{min} are strongly damped. The field δf is normalized in order to approximate the density variance to the desired value. Finally, the density field is also re-normalized in order to guarantee $\langle \rho \rangle = \rho_0$. The values $\delta\rho_{\text{rms}}/\rho_0$ shown in Table 1 are calculated from the resulting density field.

Figure 2 (top) shows the unidimensional power spectrum of the initial density fields for the models A presented in Table 1. The upper map in Figure 3 shows the initial density distribution in a central cut of the xy -plane of the box for model AI.

2.1.2 MHD turbulence density fluctuations

Density structures in the ISM are generated by the highly compressible MHD turbulence combined with cooling and gravity effects (e.g., Burkhart et al. 2009). Therefore, the density fluctuations generated by MHD turbulence represent more realistically the ISM density structures than the synthetic structures described above. It is important to compare then the system evolution of the turbulent density interacting with the CR precursor when it is artificially produced with that generated by MHD turbulence. With this aim, we also run models where the initial density structure is taken

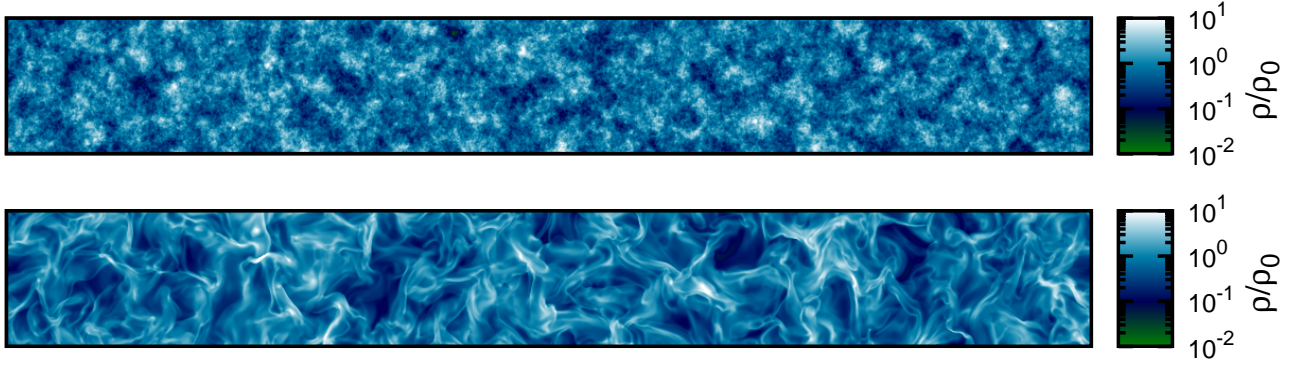
¹ We are considering a constant diffusion scale (diffusion coefficient), that attributed to the highest cosmic-ray energy; hence such a spatial exponential decay is naturally expected.

² In the models description the parameter “ \mathcal{M}_s ” refers to the value of the Mach number of the pre-shock flow at infinity (far from the shock); in the simulations it is the physical value at the inflow boundary.

³ Due to the non-vanishing numerical resistivity, MHD simulated flows have an upper limit Reynolds number. This upper value is still much lower than the real astrophysical Reynolds and magnetic Reynolds numbers.

Table 1. Models parameters

Model	λ_ρ/D_{CR}	$\delta\rho_{rms}/\rho_0$	$\mathcal{M}_{s\text{ turb}}^*$	$\mathcal{M}_{A\text{ turb}}^*$	$L \times L_y \times L_z (\div D_{CR})$	$N_x \times N_y \times N_z$
AI	0.05	0.84	-	-	$1 \times 0.125 \times 0.125$	$1024 \times 128 \times 128$
AII	0.05	0.24	-	-	$1 \times 0.125 \times 0.125$	$1024 \times 128 \times 128$
AIII	0.0125	0.84	-	-	$1 \times 0.125 \times 0.125$	$2048 \times 256 \times 256$
HAI	0.05	0.85	-	-	$1 \times 0.125 \times 0.125$	$2048 \times 256 \times 256$
BI	0.05	0.90	2.3	2.9	$1 \times 0.125 \times 0.125$	$1024 \times 128 \times 128$
BII	0.05	0.28	0.7	2.7	$1 \times 0.125 \times 0.125$	$1024 \times 128 \times 128$
BIII	0.05	0.91	2.3	0.6	$1 \times 0.125 \times 0.125$	$1024 \times 128 \times 128$
BIV	0.05	0.34	0.7	0.7	$1 \times 0.125 \times 0.125$	$1024 \times 128 \times 128$

**Figure 3.** Initial density distribution in a central cut of the xy -plane of the computational box for Model AI (upper panel) and for Model BI (bottom panel). The parameters of the models are listed in Table 1.

from data of MHD turbulence simulations (Models B in Table 1).

The data are 3D numerical experiments of compressible MHD turbulence for a range of Mach numbers listed in Table 1. The sonic and Alfvénic Mach numbers are defined respectively as $\mathcal{M}_{s\text{ turb}}^* = \langle |\mathbf{v}|/c_s \rangle$ and $\mathcal{M}_{A\text{ turb}}^* = \langle |\mathbf{v}|/v_A \rangle$, where the averaging is taken over the whole volume of the turbulence data. Both sub-Alfvénic and super-Alfvénic turbulence were considered. We produce these simulations with the appropriate geometry for our study (rectangular), using the *AMUN* code⁴ (Kowal et al. 2007). In these simulations, turbulence is driven solenoidally in a rectangular domain, using an isothermal equation of state. The snapshots used here correspond to an arbitrary time in which the turbulent cascade is already fully developed.

The initial density distribution is illustrated in the lower map in Figure 3, which shows the central xy -plane of the box for model BI of Table 1. Figure 2 (bottom panel) shows the unidimensional power spectrum of the initial density fields for the models B presented in Table 1.

For simplicity and to allow an easier comparison with Models A and with previous works, for the initial conditions the physical fields other than density are not taken from the MHD turbulence simulation data cubes.

2.1.3 Parameters choice

The fact that we keep the CR diffusion scale D_{CR} constant, which is the only physical length of the problem, allow us to parametrize the system by basically two dimensionless quantities: the density contrast $\delta\rho_{rms}/\rho_0$ and the relative density structures scale λ_ρ/D_{CR} . While the density contrasts can be inferred from observations of the ISM, the density structures size are more difficult to compare with the CR diffusion scale, because this last quantity depends on the CR diffusion coefficient κ_{CR} , which is very hard to determine theoretical and observationally.

Using for κ_{CR} the Bohm diffusion coefficient (as a lower limit) of the most energetic particles assumed to be accelerated at the shock ($E \sim 10^{15}$ eV⁵), $v_0 = 3 \times 10^8$ cm s⁻¹ and $B \sim 100$ μ G (amplified field), the precursor scale gives $D_{CR} = \kappa_{CR}/v_0 \geq \kappa_{Bohm}/v_0 \sim 10^{18}$ cm.

In the warm ISM the density contrast $\delta\rho_{rms}/\rho_0$ are of order of unity (Armstrong, Rickett & Spangler 1995). For smaller Mach numbers $\delta\rho_{rms}/\rho_0$ is expected to decrease, as found from the simulated ISM turbulence (see Table 1, and Burkhart et al. 2009). The typical outer scales of the ISM turbulence L_{Gal} extend from 1 pc in the spiral arms, up to 100 pc in the interarm regions (Haverkorn et al. 2008)⁶.

The range of λ_ρ/L is chosen in order to minimize the

⁵ This maximum CR energy requires an amplified magnetic field.

⁶ Note that for the parameters we are considering, turbulence structures present in the precursor are at scales $D_{CR}/80 < \lambda_\rho <$

⁴ <http://www.amuncode.org/>

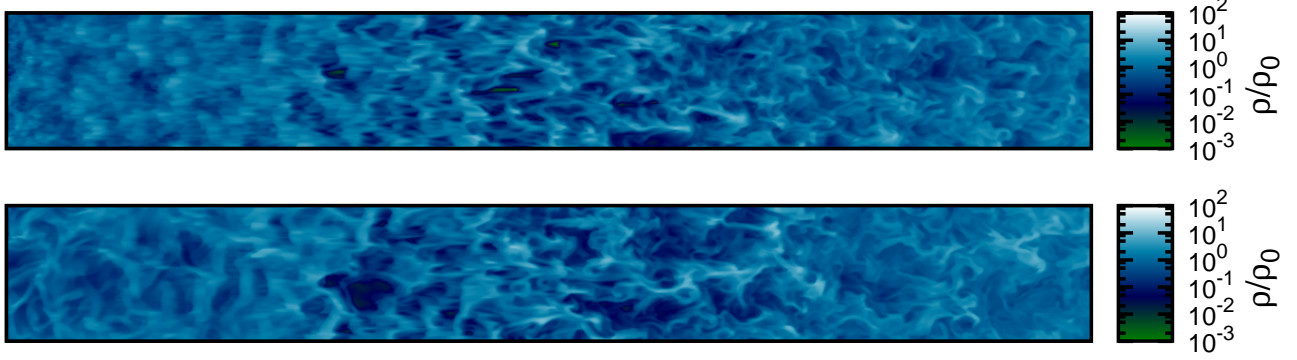


Figure 4. Final density distribution in a central cut of the xy -plane of the computational box for Model AI (upper panel) and for Model BI (bottom panel). The parameters of the models are listed in Table 1.

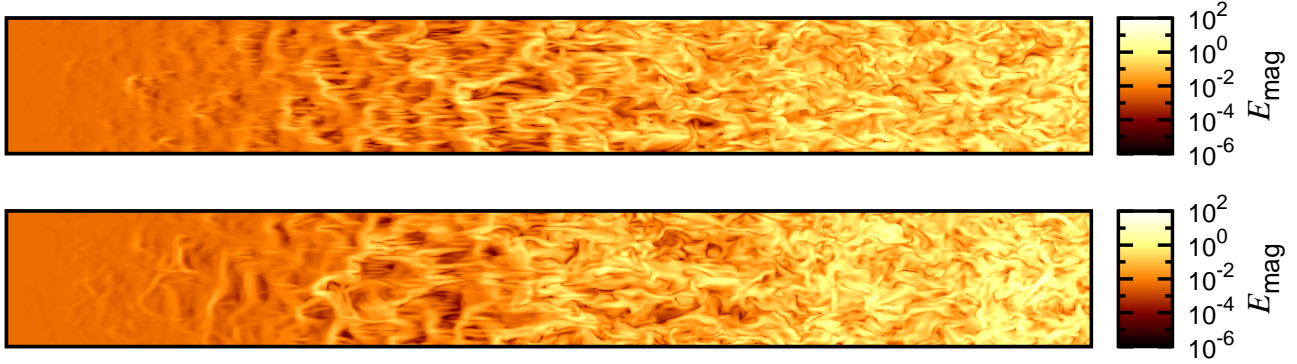


Figure 5. Final distribution of the magnetic energy in a central cut of the xy -plane of the computational box for Model AI (upper panel) and for Model BI (bottom panel). The parameters of the models are listed in Table 1.

eddy turn-over time of the induced turbulence ($\sim \lambda_\rho/v_0$), compared to the time the gas takes to cross the precursor length. This length is defined by the computational box length in the x -direction, which is the region of influence of the CR pressure gradient: $\sim D_{\text{CR}}$. The ratio between the last and the former times gives some estimative of the number of turn-over times of the largest turbulent eddies. Is expected that the larger this ratio ($D_{\text{CR}}/\lambda_\rho$), the more amplification of the magnetic field is accomplished by the turbulent dynamo before the gas crosses the shock.

Models AI and AII in Table 1 have $D_{\text{CR}}/\lambda_\rho = L/\lambda_\rho = 20$ and different values of the density contrast $\delta\rho_{\text{rms}}/\rho_0$. Model AIII has the same density contrast as model AI, but a larger value of $D_{\text{CR}}/\lambda_\rho = 80$. This model has a different resolution in order to avoid (numerical) dissipative effects to nullify the impact that a smaller value of $\lambda_\rho/D_{\text{CR}}$ has on the dynamo amplification of the magnetic field. Model HAI has the same parameters as Model AI, except for the higher grid resolution.

The various phases of the ISM exhibit different turbulence regimes, as already mentioned above (see also Section 1.1). For models B, using density structures from MHD turbulence, we consider: supersonic/super-Alfvénic

turbulence in model BI, subsonic/super-Alfvénic turbulence in model BII, supersonic/sub-Alfvénic turbulence in model BIII and subsonic/sub-Alfvénic turbulence in model BIV. The density structures produced under these different regimes are statistically different.

Our calculations use non-dimensional code units. In order to scale the quantities to physical units, we have to adopt three reference units which are equivalent to the code density, velocity, and length units. Considering typical values for the ISM, the reference physical units can be considered as follows: $\rho_{0\text{ cgs}} = 2.3 \times 10^{-22} \text{ g cm}^{-3}$ (initial mean density), $c_{s0\text{ cgs}} = 10 \text{ km s}^{-1}$ (initial unperturbed sound speed), $D_{\text{CR cgs}} = 10^{18} \text{ cm}$ (the CR diffusion scale). In this way, the initial value of the magnetic field is $B_{0\text{ cgs}} = B_{0\text{ cgs}} \sqrt{4\pi\rho_{0\text{ cgs}}} \approx B_0 \times 50 \text{ } \mu\text{G}$, where B_0 is the initial magnetic field intensity in code units.

2.2 Boundary conditions

Except for the x -boundary conditions, all boundaries are set periodic. At the left x -boundary ($x = 0$), the gas inflowing has the same physical fields from the initial conditions, mapped at $x = -v_0 t$ (and using periodicity in the x -direction). At the right x -boundary ($x = L$), outflow boundary conditions are used (null gradient across the boundary, and no inflow). However, we keep constant pressure on the

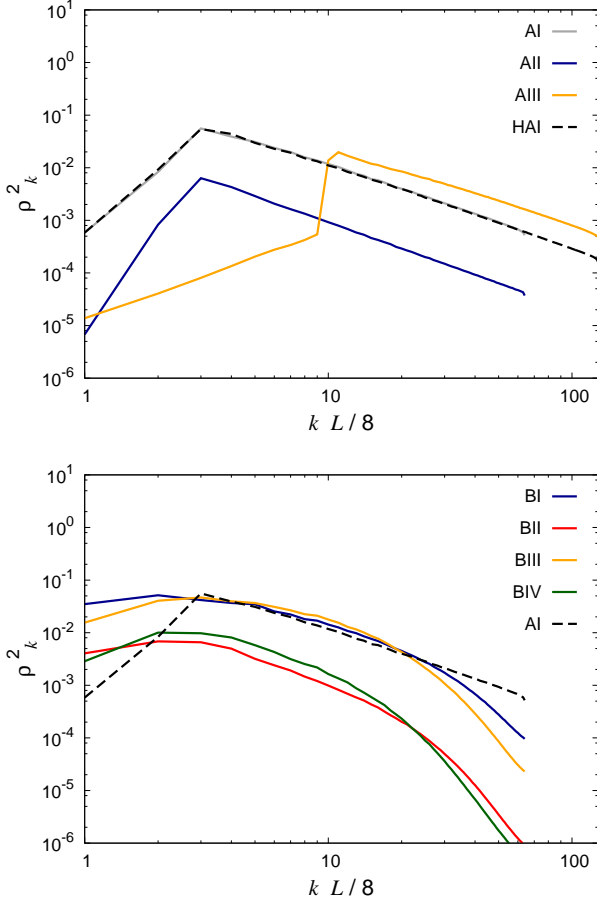


Figure 2. Power spectrum of the initial density field ρ for Models A (top) and Models B (bottom) presented in Table 1.

right x -boundary to avoid sound waves to propagate inside the domain, as reported in Drury & Downes (2012).

3 RESULTS

The system is evolved during $4L/v_0$ time units. After $t \approx 2L/v_0$, it reaches the statistically stationary state. The value of the force as a function of time is shown in Figure 6 for models AI and BI. For all the models it is nearly identical, given approximately by $f_{CR} = 0.4\rho_0 v_0^2/D_{CR}$ after the system stabilization.

We study the profiles of the magnetic field energy, density, and velocity, along the direction parallel to the shock propagation (x -axis), and the power spectrum of these fields. The power spectrum are computed in the plane parallel to the shock front (the yz -plane) at the last computational cells in the x -direction, that is, at the point closest to the shock (unless stated otherwise). These quantities are calculated for each snapshot from $t = 3L/v_0$ to $t = 4L/v_0$ (taken between fixed intervals $\Delta t = 0.2L/v_0$) and then averaged.

3.1 Profiles

As explained in Section 1, according to BJJ09 the interaction of density fluctuations with the CR precursor produces

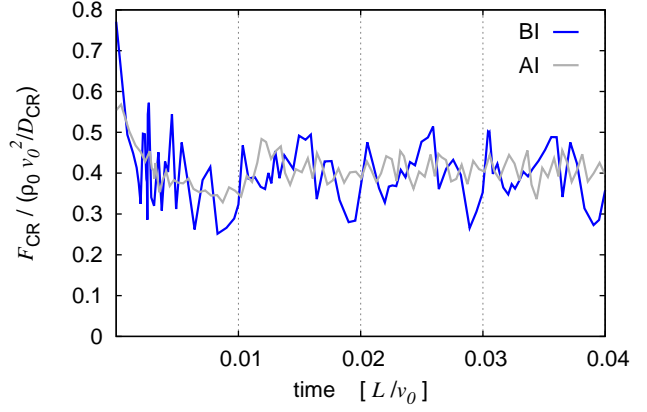


Figure 6. Time evolution of F_{CR} at the right boundary for models AI and BI.

further turbulence partially solenoidal (that is, vorticity) which amplifies the magnetic field through the small-scale dynamo.

Figure 7 shows the profile of the vorticity intensity ω for the models in Table 1. The comparison between models AI and AII (top panel) shows that vorticity is generated at a faster rate with distance (along the x -direction) for models with the higher values of $\delta\rho_{rms}/\rho_0$. Comparing models with the same density contrast but different density structures size (models AI and AIII), the vorticity increases with distance more quickly for the model with smaller λ_ρ . We also observe that the vorticity intensity ω , after a phase of fast increase in the beginning of the box, saturates (or increases very slowly) for the models with the highest values of $\delta\rho_{rms}/\rho_0$. The models with simulated turbulence (bottom panel) show the same behaviour as models A (model AI is shown in dashed line for comparison), with the rate of vorticity generation being higher for the models with higher density contrast (models BI and BIII).

The magnetic energy profiles are shown in Figure 8. We can infer for models A (top panel) that the rate of magnetic energy growth per distance is larger for models AI and AIII; models with different density structure sizes also show differences in the magnetic field evolution: it is faster for the models with smaller λ_ρ at the beginning of the box, but the magnetic energy achieves a maximum and then saturates before the end of the box is reached. The final magnetic energy (at $x = L$) differs greatly between models AI and AII, but it is similar between models with different values of λ_ρ (AI and AIII). Models AI and AII differ in $\delta\rho_{rms}/\rho_0$ by a factor of 3, and the final magnetic energy density increases by a factor of ≈ 115 in the first case and ≈ 39 in the second one. The same dependence of the magnetic energy growth rate on $\delta\rho_{rms}/\rho_0$ is observed in models B (Figure 8, bottom panel).

Part of the magnetic field amplification observed in Figure 8 is due to plasma compression. As the CR pressure slows down the incoming flow in the precursor the plasma is compressed by a certain factor, as a consequence the magnetic field (which is initially perpendicular to the shock) increases its intensity by approximately this same factor. Figure 9

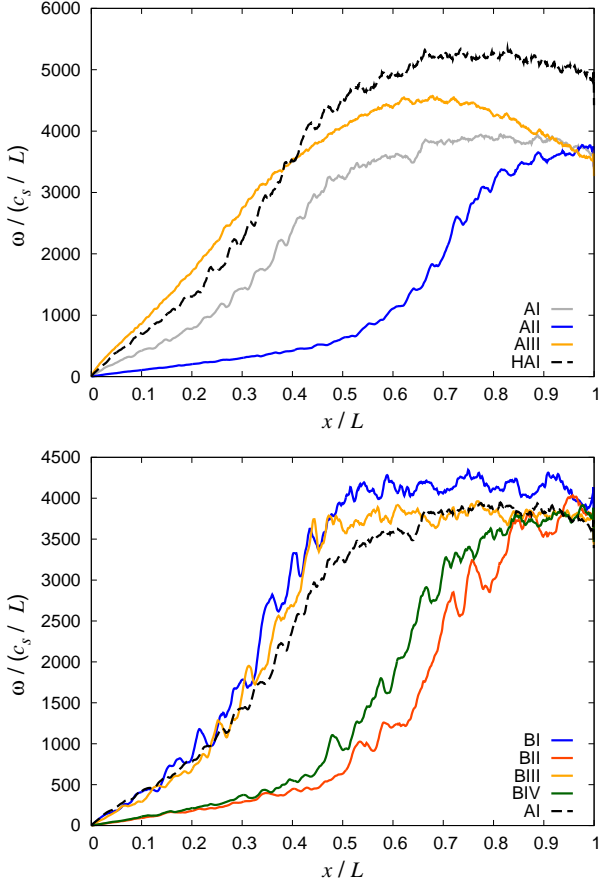


Figure 7. Profile of the *rms* vorticity (averaged in the yz -plane) for models A (top) and models B (bottom). The profiles were averaged in time from $t = 3L/v_0$ to $t = 4L/v_0$.

shows the profiles of the mean velocity for models A (top panel) and B (bottom panel).

The initial velocity is reduced by $\approx 30\%$ of its initial value in all models; at the same time that density is increased by a similar factor (not shown). Therefore, the compression can account only for an increase of a factor ≈ 1.7 in the magnetic energy density, and the remaining being due to the small-scale dynamo.

A key ingredient in the BJL09 theory is the generation of solenoidal motions within the precursor, which is the turbulent velocity component responsible for amplifying the magnetic field. In order to separate the solenoidal component \mathbf{v}_s ($\nabla \cdot \mathbf{v}_s = 0$) of the velocity field from its curl-free component \mathbf{v}_p ($\nabla \times \mathbf{v}_p = 0$), we calculate the potential fields ϕ and \mathbf{A} from the the Helmholtz decomposition:

$$\mathbf{v} = \nabla\phi + \nabla \times \mathbf{A}, \quad (10)$$

with $\nabla\phi = \mathbf{v}_p$ and $\nabla \times \mathbf{A} = \mathbf{v}_s$.

Top row of Figure 10 shows the profile of the *rms* of the velocity fluctuations $\delta\mathbf{v}_s$ for models A (left) and models B (right). Comparing models AI and AII, we clearly see that the solenoidal velocity fluctuation is higher for model AI, with higher $\delta\rho_{rms}/\rho_0$. In the middle of the box ($x/L = 0.5$), the solenoidal fluctuations for these models are $\approx 1 c_s$ (AII) and $6 c_s$ (AI), although at the end of the box ($x = L$) the velocities are approximately equal ($\approx 6 c_s$). The solenoidal velocity of model AII initially increases more slowly and then

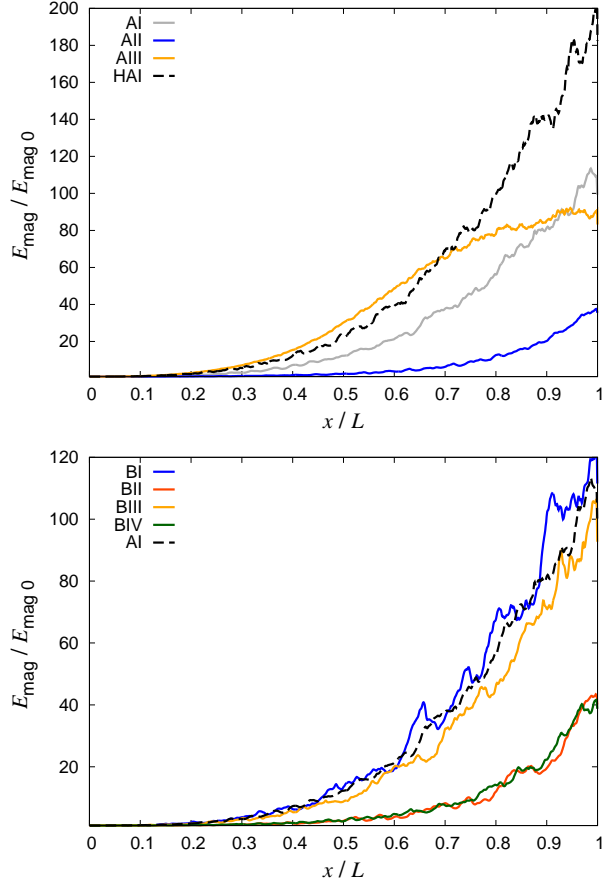


Figure 8. Profile of the magnetic energy density for models A (top) and models B (bottom). The profiles were averaged in time from $t = 3L/v_0$ to $t = 4L/v_0$.

increases at a higher rate (after $x/L = 0.5$). The model AIII, with smaller λ_p/D_{CR} , on the other hand, presents slower growth of velocity fluctuations after the middle of the box is achieved. The models B (right column) show a trend similar to that of models A in the growths of solenoidal velocity fluctuations: an initially slow growth followed by a faster growth for models with smaller $\delta\rho_{rms}/\rho_0$ (models BII and BIV). The models BI and BIII show a fast increase until $x \approx 0.5 - 0.6L$, where the maximum is achieved; after that a fast decrease by a factor of approximately 2 is followed, to finally decrease slowly at the end of the box.

In the middle row of Figure 10, the profiles of the *rms* of the potential velocity fluctuations $\delta\mathbf{v}_p$ are shown. The potential fluctuations achieve nearly the same saturation level $\delta v_{p,rms} \approx 2 c_s$ for models AI and AII. For model AII, with smaller $\delta\rho_{rms}/\rho_0$, the saturation level is only achieved at larger values of x ($x \approx 0.7$). The model AIII with smaller λ_p , on the other hand, has lower maximum values of the potential fluctuations, compared to the reference model AI (≈ 2.5 for model AI, and ≈ 1 for model AIII). Once more models B (right column) follow similar trends as models A, with the density contrast ruling the behaviour of the models.

The potential velocity fluctuations are important to generate more density structures along the preshock, otherwise the solenoidal turbulent motions will operate only in a small fraction of the preshock, because besides amplifying

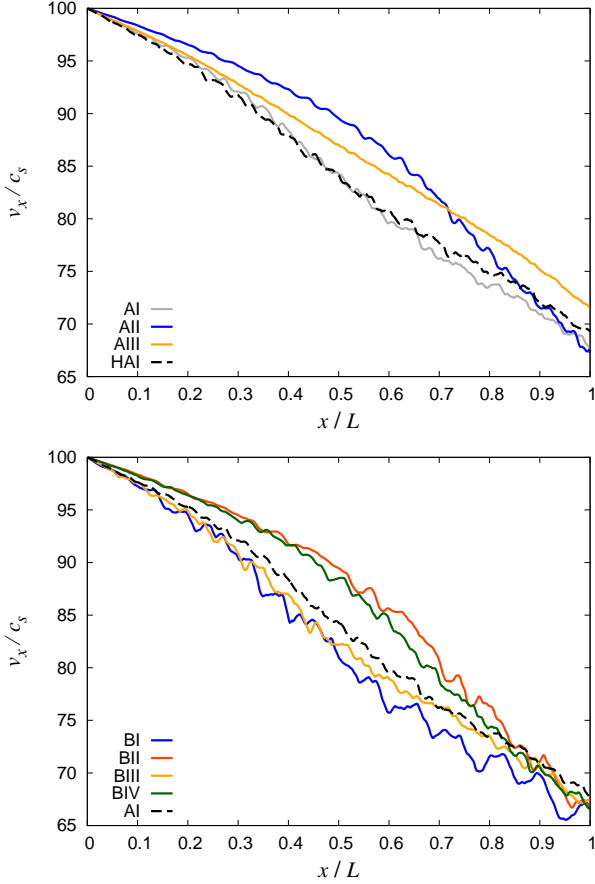


Figure 9. Profile of the x -component of the velocity for models A (top) and models B (bottom). The profiles were averaged in time from $t = 3L/v_0$ to $t = 4L/v_0$.

the magnetic field, the solenoidal turbulent motions act to homogenize the density field (due to turbulent mixing). The last row of Figure 10 shows the profiles of the *rms* of the density fluctuations $\delta\rho$. Comparing models A (left column), it can be seen that after an interval in which the density fluctuations remain constant (until $x/L \approx 0.3$ for model AII), they grow at nearly the same rate for models AI and AII, and after achieving a maximum value, the fluctuations start to decrease. The maximum values are higher and are achieved first for models with higher initial density contrast. In the case of model AIII, after the maximum value is achieved in the density contrast at $x/L \approx 0.5$, it diminishes. This maximum value is smaller as smaller the λ_ρ , and it should be remarked that the values of the density fluctuations at the end of the box are smaller than the initial values, meaning that density fluctuations are not efficiently generated by the induced turbulence, in agreement with the potential velocity fluctuations (middle row of Figure 10). Figures 4 and 5 show the density and magnetic energy density distribution, respectively, in the central xy -plane for models AI and BI, at the time $t = 4L/v_0$.

3.2 Power spectrum

The turbulent dynamo, usually studied in the incompressible limit of the MHD equations, amplifies the magnetic

field energy from the smaller scales to the larger scales (see Beresnyak & Lazarian (2015) for a recent review). The saturation is expected to happen scale-by-scale (first at the smaller scales), when the magnetic energy achieves near equipartition with the kinetic energy. For the incompressible reference case, it is observed that in the saturation of the dynamo, the power spectrum of the magnetic field (magnetic energy) is close to the velocity field power spectrum (kinetic energy).

Figure 11 shows a comparison between the power spectrum of the magnetic field \mathbf{B} with the power spectrum of the solenoidal velocity field \mathbf{v}_s , for models A (top panel) and models B (bottom panel). The power spectra are calculated in the yz -plane at the last cell in the x -direction of the domain. The Kolmogorov power law $\propto k^{-5/3}$ is also shown for comparison. We observe that for all A models the solenoidal velocity field power spectrum peaks approximately at the wave-number corresponding to the maximum of the initial density structures ($k_{\max} \approx L_y/\lambda_\rho$). At these scales, the magnetic power spectrum for all models A is approximately 2 orders of magnitude lower than the solenoidal velocity power spectrum. Model AII presents magnetic field power spectrum smaller than those of model AI. Model AIII, when compared to model AI, presents the power spectrum of the magnetic field closer to the power spectrum of the solenoidal velocity component, that is, closer to the equipartition at the smaller scales. Model B (bottom panel) presents the power spectrum for the magnetic field lower for models with lower density contrast. However, contrary to models A, their power spectra present more power at the larger scales (smaller k), especially for models BI and BIII. The difference between the magnetic and velocity power spectra is about 1 order of magnitude for larger values of k ($k > 20$); it is approximately 2 orders of magnitude for smaller values of the wave-number ($5 > k > 10$), and the maximum difference of about 3 orders of magnitude is reached for $k = 1$.

Figure 12 shows the power spectrum of the potential velocity field (top row) and of the density (bottom row). Comparing the power spectrum of the potential velocity field of models AI and AII, can be seen that they are nearly identical, peaking at the same wavenumber as the solenoidal velocity field does ($k \approx 4$). Model AIII differs from model AI in the power which is smaller for $k < 10$, and nearly constant. In the case of models B (left column), for $k > 2$, they have similar potential velocity spectra.

Comparing models AI and AII we see that the density power spectrum (left column in Figure 12) close to the shock are similar; the power increases at the larger scales for the model with smaller initial $\delta\rho_{rms}/\rho_0$ (AII). The peak in the final power spectra seems to be localized close to the wave-number where the initial density spectra peaks $k_{\max} \approx L_y/\lambda_\rho = 2.5$. We also observe that the power at the smaller scales decreases. This is probably attributed to the numerical dissipation, which dumps the velocity fluctuations and consequently the density fluctuates over these scales. Now comparing models AI and AIII, we do see the final spectrum of model AIII to increase for scales larger than λ_ρ ($k < k_{\min}$). All models B present a very similar final density power spectra.

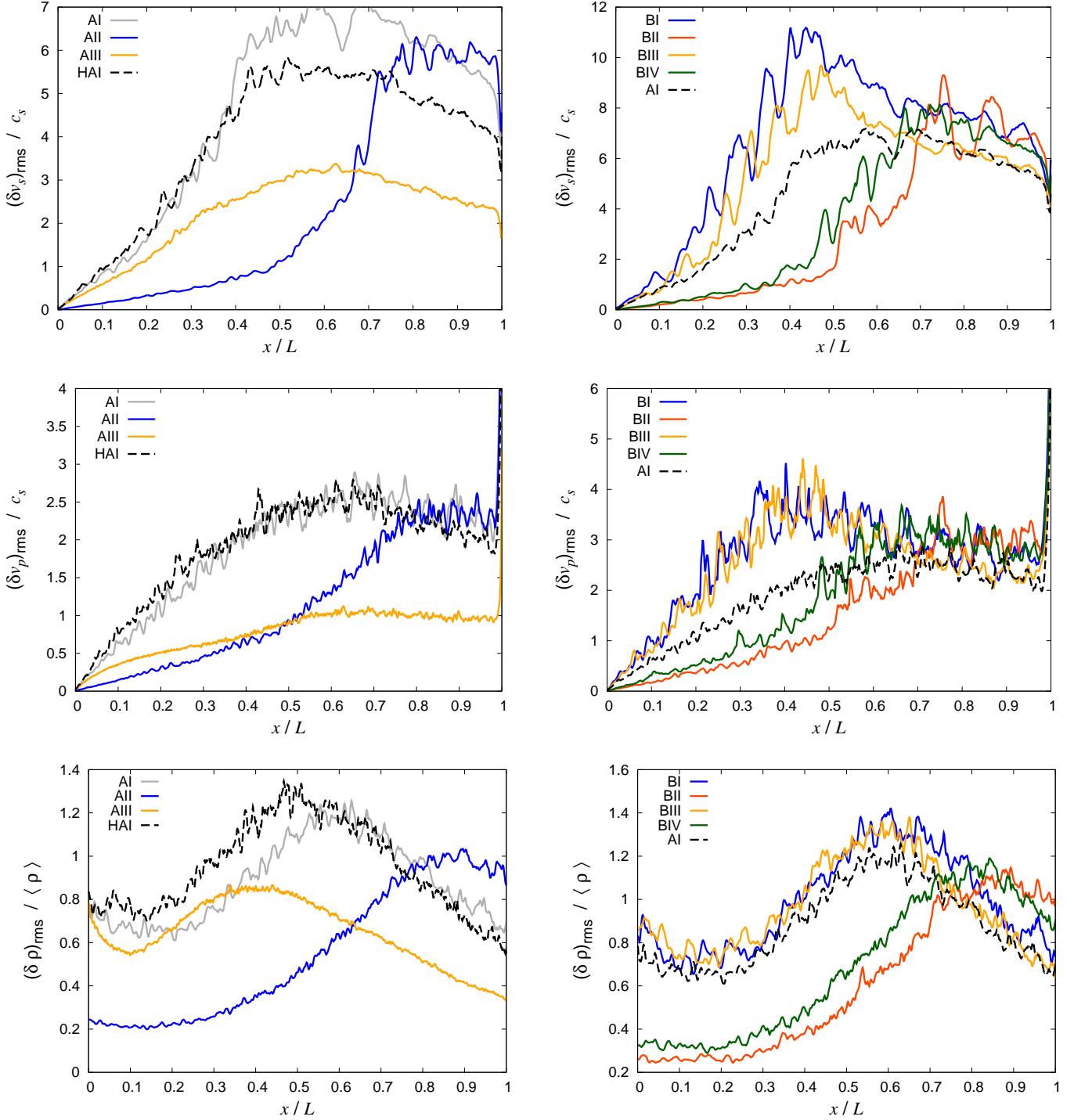


Figure 10. Profile of the *rms* solenoidal velocity fluctuations δv_s (top row), the *rms* potential velocity fluctuations δv_p (middle row), and density fluctuations $\langle \delta \rho \rangle / \langle \rho \rangle$ (bottom row). Left column: models A; right column: models B. The profiles were averaged in time from $t = 3L/v_0$ to $t = 4L/v_0$.

4 DISCUSSION

The comparison between the power spectrum of the solenoidal velocity and of the magnetic field shows that, for all the parameters considered in this work, the resulting amplification of the magnetic field close to the shock is far from equipartition with the kinetic energy available from

the solenoidal velocity field, even for the smaller scales solved in our simulations (Figure 11). The evolution of each power spectrum (solenoidal velocity and magnetic field) along the distance inside the precursor is shown for model AI in Figure 13. It can be seen that the velocity power is increasing inside the precursor at approximately the same rate for all the wavenumbers.

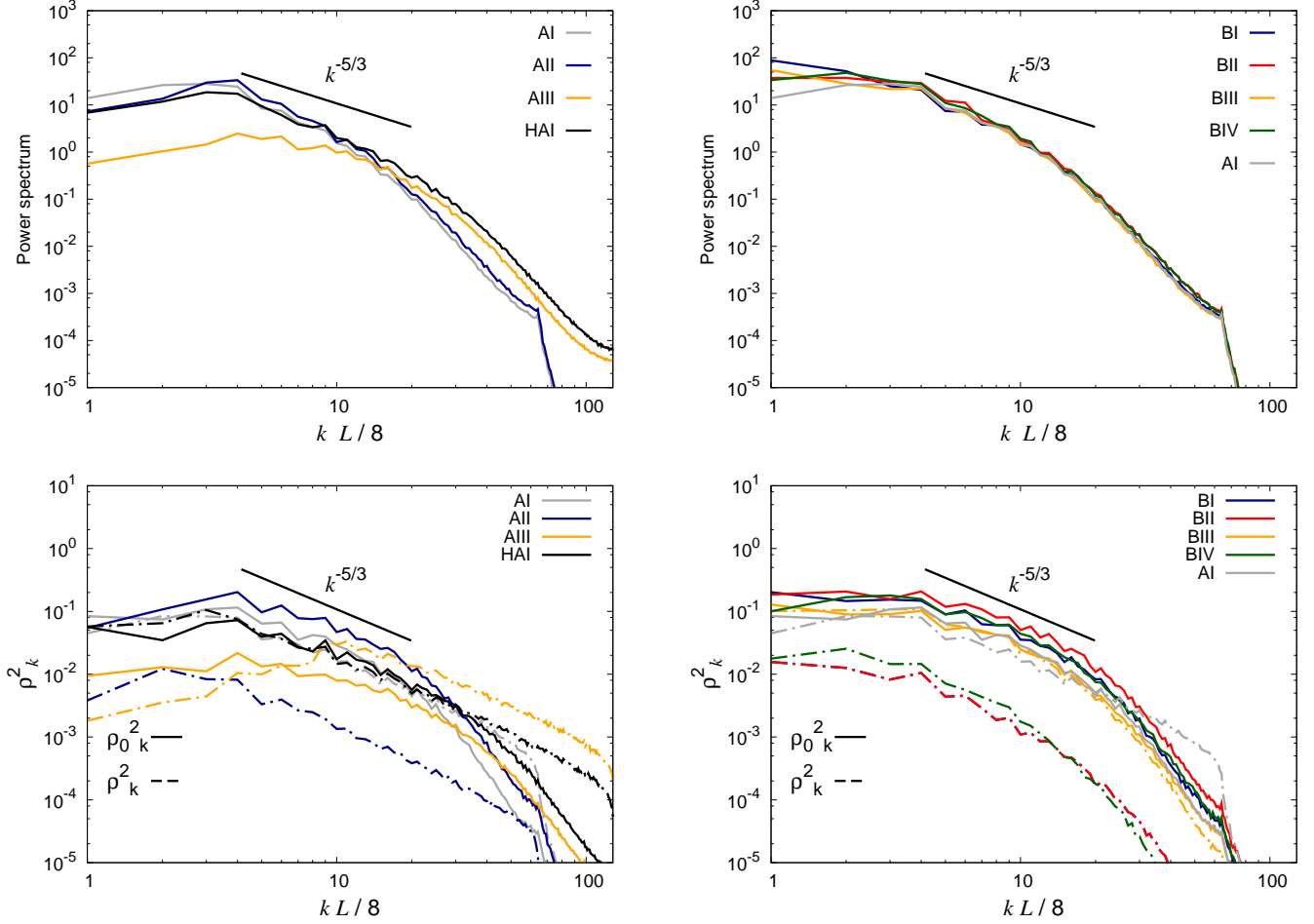


Figure 12. Power spectrum of the potential velocity field \mathbf{v}_p (top row) and of the density field ρ (bottom row) for models A (left column) and models B (right column). The power spectrum were calculated in the yz -plane of the last grid position in the x -direction. The density power spectra of the first grid position is shown for comparison. The power spectrum were averaged in time from $t = 3L/v_0$ to $t = 4L/v_0$.

The role of the potential (compressible) component of the velocity field in the turbulent amplification of the magnetic field is not clear, as the dynamo studies usually consider the incompressible limit of the MHD equations. Despite of this, the solenoidal velocity fluctuations dominate over the compressible ones in the turbulence generated (Figure 10).

Additionally, the total velocity fluctuations achieves only a few percent of the upstream flow velocity v_0 , and its evolution is shown here to depend strongly on the initial density contrast, as well as on the density structures size (Figure 10). The saturation values are in the range $0.04 - 0.1v_0$ for the models we present, and are higher for the highest values of the initial density contrast and of the density structures size.

The scales in which the CR pressure induces turbulence are shown to be concentrated in the typical scale of the density fluctuations, as expected from the BJL09 theory, at least while the density field spectrum is not modified by the precursor turbulence itself. Figure 14 compares the power spectrum of the density and velocity field in different yz -planes inside the precursor for model AIII. The peaks of both power

spectrum are approximately at the same wavenumbers, going to larger scales along x .

Therefore, it is clear that turbulence has not enough time to fully develop before the material reaches the shock, in the sense that the turnover time of the largest eddies inside the precursor is larger or comparable to the time it takes for the material to cross the precursor (however this time is not fixed, as the turbulent velocity increases along the precursor).

It could be thought that reducing the density structures size could give more time to the turbulence cascade to develop, increasing the efficiency of the small-scale dynamo. In fact, for model AIII the power spectrum of the magnetic field is closer to the solenoidal velocity field at the smaller scales, although it happens already for scales where the numerical dissipation is important; therefore, we can not exclude the possibility that this would be different for different grid resolutions (or different numerical dissipations). However, our model AIII show that the turbulent velocity increases more slowly and saturates at lower values in the precursor when comparing with model AI (see Figure 10), at the same time that the maintenance of the density structures is degraded,

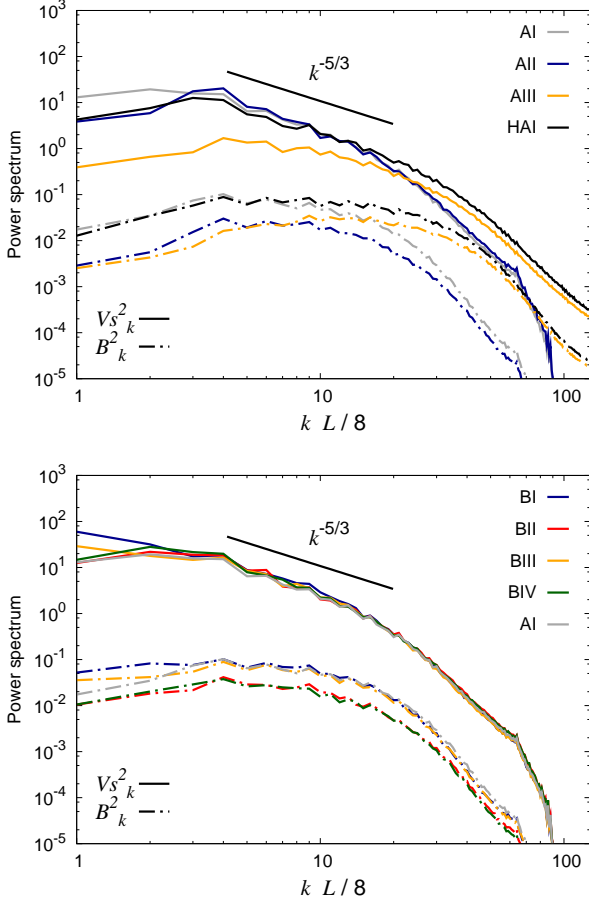


Figure 11. Power spectrum of the magnetic field \mathbf{B} (dashed lines) and of the solenoidal velocity field \mathbf{v}_s (continuous line) for models A (*top*) and models B (*bottom*). The power spectrum were calculated in the yz -plane of the last grid position in the x -direction. The power spectra were averaged in time from $t = 3L/v_0$ to $t = 4L/v_0$.

despite that the turbulent heating of the gas along the precursor decays, increasing its compressibility. Figure 15 shows the comparison of the sound speed profiles between the models studied.

This result shows that the injection of turbulent energy in the system by the CR pressure is comparatively larger when the density structures have larger sizes. Figure 16 compares the profile of “partial” kinetic energy between models, calculated taking into account only the components y and z of the velocity field.

Therefore, we can conclude that the magnetic energy amplification is larger for larger values of the initial density contrast and larger λ_ρ/D_{CR} . However, this efficiency can not grow indefinitely, because the sound speed of the gas in the precursor increases as well, decreasing the compressibility of the gas (and consequently the maintenance of the density structures) and the effective Mach number of the shock, reducing the efficiency of the CR acceleration in the shock (not considered in our formulation).

The fact that the magnetic energy amplification is less efficient as smaller the values of the injection scale λ_ρ are can be understood in terms of the mixing time of turbulence

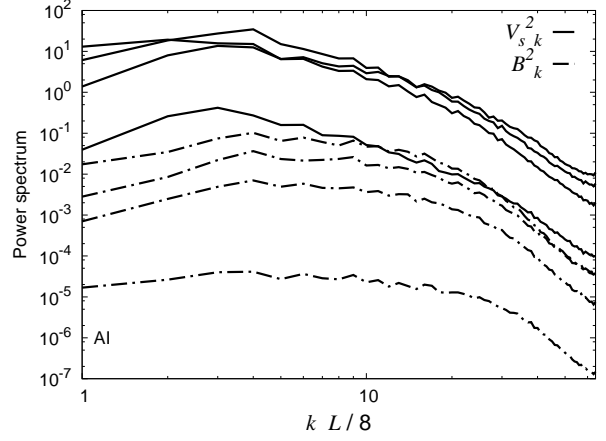


Figure 13. Power spectrum of the magnetic field \mathbf{B} (dashed lines) and of the solenoidal velocity field \mathbf{v}_s (continuous line) for model AI. Each curve represents the power spectrum calculated in the yz -plane of a different position in the x -direction. The lowermost curves are for position $x/L = 0.1$, and the uppermost correspond to $x/L \approx 1$. The intermediary curves are separated in the x coordinate by $\Delta x/L = 0.3$. The power spectra were averaged in time from $t = 3L/v_0$ to $t = 4L/v_0$.

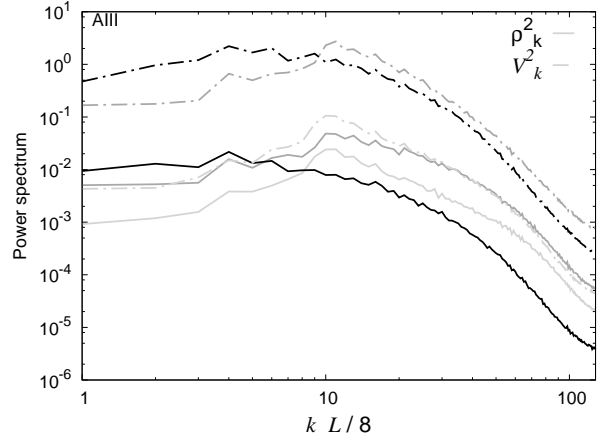


Figure 14. Power spectrum of the density field ρ (dashed lines) and of the velocity field \mathbf{v} (continuous line) for model AIII. Each curve represents the power spectrum calculated in the yz -plane of a different position in the x -direction. The curves correspond to position $x/L = 0.1, 0.55$ and 1 (from the lighter to the darker grayscale). The power spectrum were averaged in time from $t = 3L/v_0$ to $t = 4L/v_0$.

t_{mix} and the generation time t_{gen} of the turbulence-induced density structures. The former can be estimated as $t_{\text{mix}} \sim \lambda_\rho^2/\nu_{\text{turb}}$, where ν_{turb} is the eddy diffusivity, that can be estimated as $\lambda_\rho v_s$, then $t_{\text{mix}} \sim \lambda_\rho/v_s$. t_{gen} can be estimated as λ_ρ/v_p . Because $v_s > v_p$ then $t_{\text{mix}} < t_{\text{gen}}$. For smaller values of λ_ρ , this condition saturates sooner, reducing the magnetic field amplification efficiency. Turbulent heating is not responsible of saturating the density structures as can initially be thought.

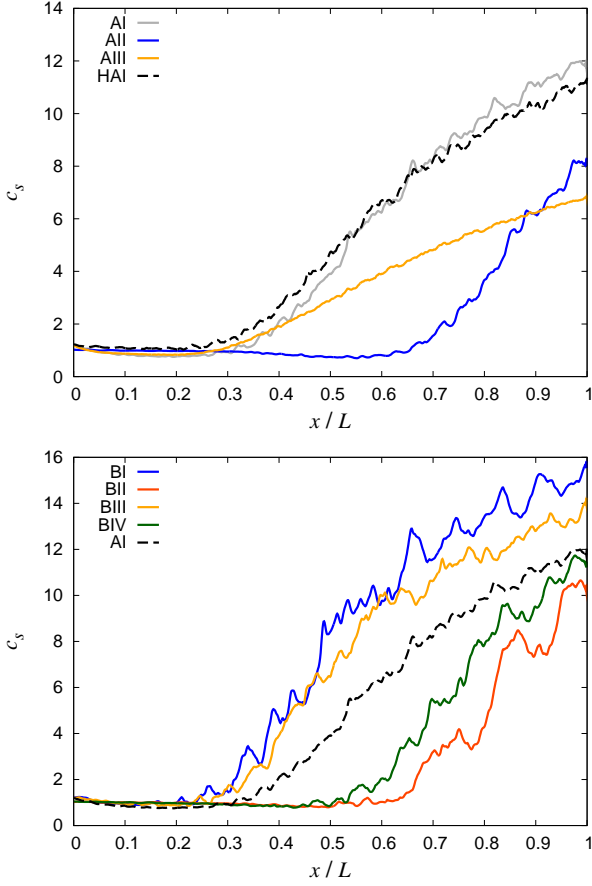


Figure 15. Profile of the sound speed $\langle c_s \rangle = \langle \sqrt{\gamma P / \rho} \rangle$ for models A (top) and models B (bottom). The profiles were averaged in time from $t = 3L/v_0$ to $t = 4L/v_0$.

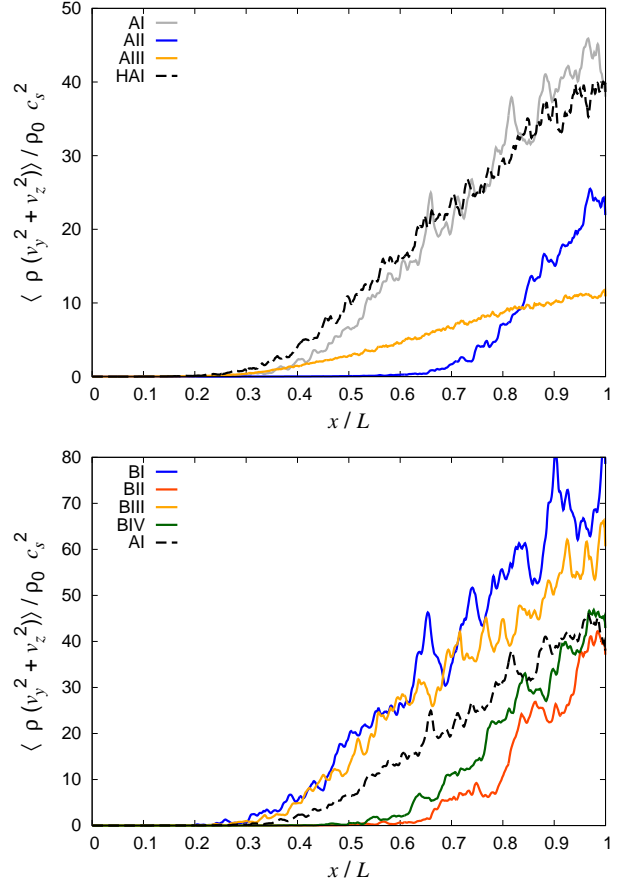


Figure 16. Profile of the partial kinetic energy $\langle 0.5 \rho (v_y^2 + v_z^2) \rangle / \rho_0 c_s^2$ for models A (top) and models B (bottom). The profiles were averaged in time from $t = 3L/v_0$ to $t = 4L/v_0$.

4.1 Resolution effects

The fact that a considerable part of the magnetic energy generated occurs at the smallest scales of the simulations makes the numerical resolution important for the results. In fact, not only the numerical dissipation of the magnetic field but also the numerical dissipation of the velocity (and also density) structures in these small scales cause a reduction of the magnetic energy amplification.

The model HAI presents a faster growth rate of vorticity with distance, compared to model AI; but the former presents a saturation value while the last increases continuously until the shock position (see Figure 7). The final magnetic energy density for model HAI is approximately twice the value for the model AI ($B/B_0 \approx 200$ for model HAI and ≈ 100 for model AI, according to Figure 8). However, the profiles for the fluctuations of the solenoidal and potential velocity, and for the density, do not differ much between them, as shown in Figure 10, which have the values dominated by the fluctuations at the larger scales (at the scales $\sim \lambda_\rho$). The power spectrum of the magnetic field shows the larger power for model HAI at the smaller scales (see Figure 11).

These results show that the magnetic energy amplification obtained in the present simulations can be underestimated, and should be considered as a lower limit.

4.2 Synthetic versus MHD turbulence density fluctuations

The comparison between the results from models A and B shows that the statistical details of the density fluctuations have little influence on the final magnetic field amplification. In the setup employed in this study, the density structures inflowing in the precursor lose their initial characteristics in a short distance, being dominated by the density structures induced by the compressional turbulence. In order to illustrate this point, Figure 17 compares the evolution of the density spectrum along the precursor for models AII and BII, which have similar initial values of $\delta \rho_{rms}/\rho_0$ and λ_ρ/D_{CR} . We see that after $x/L \approx 0.5$ both spectrum present similar shapes.

However, it should be observed that the initial density spectrum of models A and B differ on scales larger than λ_ρ , with more power in models B. The same occurs for the magnetic field spectrum close to the shock position, as can be seen comparing the models in Figure 11. Models with similar initial values of $\delta \rho_{rms}/\rho_0$ and λ_ρ/D_{CR} but slightly different spectral slope (models AI and BI, AII and BII) present similar values of magnetic field amplification (see Fig. 8).

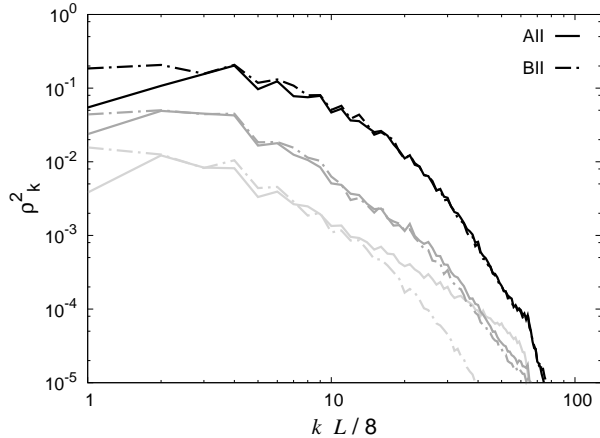


Figure 17. Power spectrum of the density field ρ for models AII (continuous lines) and for model BII (dashed lines). Each curve represents the power spectrum calculated in the yz -plane of a different position in the x -direction. The curves correspond to position $x/L = 0.1, 0.55$ and 1 (from the lighter to the darker grayscale). The intermediary curves are separated in the x coordinate by $\Delta x/L = 0.45$. The power spectra were averaged in time from $t = 3L/v_0$ to $t = 4L/v_0$.

4.3 Dependence on the shock Mach number

We ran an additional model using the same parameters as model AI (see Table 1), but with $\mathcal{M}_s = 400$. The resulting CR pressure is increased approximately by a factor ~ 16 , and consequently the turbulent energy is greater in this system. However, there is not enough time for the magnetic field to grow further, and the final magnetic field is almost the same as in the original case with $\mathcal{M}_s = 100$.

Due to our self-regulated formulation, we do not expect great differences in the magnetic field amplification with the shock Mach number (we are not considering the dependences that the particle acceleration efficiency might have with \mathcal{M}_s); although a very different value might produce differences in the magnetic field amplification. However we are interested in strong non-relativistic Galactic shocks, in which the Mach numbers do not differ significantly.

4.4 Limitations of the present study

The limited domain size in the direction parallel to the shock propagation might reduce the magnetic field amplification. We considered the CR pressure acting only inside the scale $L = D_{CR}$, which means that the effects of the force at the distances where it decayed by a factor smaller than ≈ 3 is neglected. In order to test this we made a run with the initial conditions given by model AI but doubling the box dimension in the x -direction, i.e. $L = 2D_{CR}$, and resolution $2048 \times 128 \times 128$. The final magnetic field in this case is approximately twice the value obtain for model AI (not shown).

Less important, if the computational box in the perpendicular directions (y and z) were larger, some small amplitude density fluctuations with scales larger than λ_ρ could develop, leading to additional magnetic field amplification. Also the presence of cooling could increase the genera-

tion of higher density fluctuations. Our models with higher initial density contrast have also higher magnetic field amplifications. However, the presence of cooling was considered in Downes & Drury (2014) (see next section), and they did not detected a substantial increase on the magnetic energy amplification.

As mentioned in Section 2.1.2, we do not consider the turbulent fluctuations of velocity and magnetic field in the initial conditions of models B; this can be justified taking into account that the real ISM density structures are formed in the presence of cooling; therefore the density fluctuations are not totally dependent on the presence of magneto-acoustic modes, and can be present even after the decays of velocity and magnetic turbulence fluctuations (entropy modes). It is interesting to note that the velocity and magnetic fluctuations present from the MHD turbulence simulations have larger amplitudes than those generated by the CRs pressure. Hence, if an initial turbulent magnetic field component is considered for the initial state (in the upstream material), the magnetic field amplification close to the shock will be lower compared to our results.

Finally, we do not consider the CR pressure P_{CR} in the total pressure P (see Eqs. (3)-(7)). Adding P_{CR} to the total pressure might lead to a more incompressible plasma, hence the density fluctuations might saturate faster, possibly reducing the magnetic field amplification.

4.5 Comparison with previous studies

The first numerical study exploring the magnetic field amplification in the shock upstream via MHD turbulence induced by CR shock precursor was presented in Drury & Downes (2012). Employing 2D simulations for a shock propagating perpendicular to the initially uniform magnetic field, they studied the magnetic field profile inside the precursor for different amplitudes of the initial density fluctuations, and also for different values of the initial magnetic field intensity. They found that the final magnetic energy amplification factor depends little on the amplitude of the initial density fluctuations. It was interpreted as resulting from non-linear effects of turbulence, which increases the density contrast along the precursor and then reduces the influence of the initial values. They also found that the magnetic field amplification does not depend on its initial value, as it could be expected if the magnetic energy is far below the equipartition.

Later on, Downes & Drury (2014) investigated the influence of the initial magnetic field orientation, the presence of cooling, and more importantly, they performed three-dimensional numerical simulations. They showed that the resulting magnetic field profile in the 3D setup is very similar to that of the 2D one, with a slightly smaller magnetic field amplification in the former case. The magnetic field power spectrum are clearly different in the 2D and 3D cases as expected: in the 3D setup the magnetic field structures have smaller sizes; this difference arises because the velocity cascade of 2D turbulence proceed as an inverse cascade (as in hydrodynamic turbulence), 2D turbulence is not appropriate for modelling MHD turbulence. Concerning the initial magnetic field orientation, they found that larger amplification is produced in the perpendicular case. However, this difference in amplification with the parallel case can be ex-

plained, as showed in the paper, to be produced by compression. Finally, they showed that for the density scales they considered (representative of the ISM) cooling has little impact on their 2D simulations, because the cooling time is longer than the time the gas takes to cross the precursor.

Similar to the study in Downes & Drury (2014), the magnetic field amplification in our simulations does not converge; we also attribute this to the fact that amplification along the precursor is still occurring at scales below the scales that can be resolved in our simulations. In fact, the solenoidal velocity is also continuously increasing along the precursor, although after passing the middle of the box (in the x direction) the power spectrum shape remains approximately unchanged until the end of the domain (see Fig. 14). We have no evidences that the small-scale dynamo in any of our simulations enters the linear phase after equipartition is already achieved at some scale. If the dynamo is still in the exponential growth phase, it can help to explain the similarities between the 2D and 3D amplifications found in Downes & Drury (2014), because the turbulence cascade is still not regulating the amplification of the field in the 3D case. We have inferred that the turn-over time of the turbulence outer scale in our setups is not short compared with the time these eddies remain inside the precursor.

When studying the role of cooling, Downes & Drury (2014) focused on their 2D models. It is expected that in the 3D case, where the forward cascade quickly heats the gas reducing its compressibility and then the generation of density structures, the cooling to have a stronger impact. Another important point is related to the generation of CRs in the shock. The acceleration process is though to be sensitive to the sonic Mach number of the shock. Therefore the turbulence heated gas in the precursor also has the effect of reducing the CR pressure, which in turn reduces the magnetic field amplification in the precursor. However, the influence of the precursor on the shock evolution itself is not considered in our simplified model, which requires a much more involved modelling of the problem.

A difference between our approach and that in Drury & Downes (2012) and Downes & Drury (2014) is in the model of the CR pressure in the precursor. In our approach, the CR pressure is considered to decay exponentially (instead of linearly) from the shock front, and consequently the CR force also decays exponentially. Although the self-consistent profile of the CR pressure is difficult to calculate — as it depends on the energy of the CR rays considered and their diffusivity which in turn is anisotropic and depends on the physics of the upstream flow (such as the magnetic fields fluctuations, the velocity profile, etc.) — we just considered the solution for the CR energy in the diffusion approximation for a fixed value of the diffusivity, and without considering the influence of the CR pressure on the upstream flow.

Another difference in the CR pressure modeling, and probably the most relevant, is the use of the flow variables close to the shock in order to obtain the ram pressure of the flow at the shock. The CR force amplitude is updated dynamically in our simulations, accordingly to the kinetic energy of the upstream flow available at the shock. The previous works considered a fixed force $F_{CR} = -\eta\rho_0 v_0^2/D_{CR}$, which can lead to non-physical situations (no shock) if the induced turbulence in the precursor increases the sound

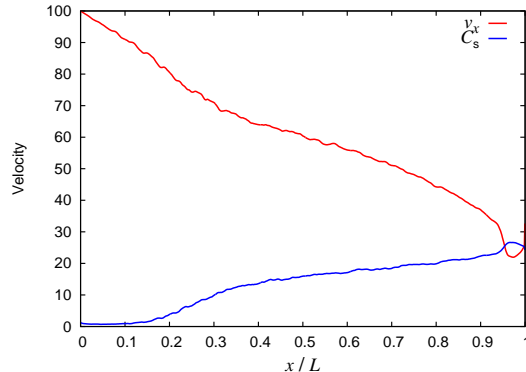


Figure 18. Profiles of the flow velocity and of the sound speed $c_s = \sqrt{\gamma p/\rho}$. The simulation uses a CR force in the form $F_{CR} = -\eta\rho_0 v_0^2$.

speed in the gas above the upstream velocity, making the problem ill-posed; this can be seen in Fig. 18, where the fluid velocity versus sound speed profiles are shown for a simulation with a constant CR force. The plasma reaches temperatures so high that the flow velocity gets lower than c_s (no shock) and then no Fermi acceleration would be operating.

Our formulation imposes a more stringent limit on the efficiency of the magnetic field amplification mechanism, which we believe to be more realistic: a stronger force reduces more the pre-shock velocity flow, at the same time more turbulence is generated; this reduces the upstream flow ram pressure and hence the CR acceleration becomes less efficient, reducing the CR pressure force. The system is self-regulated. From Fig. 19, can be seen that in the case of a constant force used in the previous works the flow ram pressure at the boundary is less than one third the value used $\rho_0 v_0^2$ for the CR force calculation, that is, the amplitude considered for the CR force is overestimated.

5 SUMMARY AND CONCLUSIONS

In this work we investigated turbulence-induced magnetic field amplification in shock precursors under the BJL09 model. We used MHD simulations to quantify the problem, modelling the CR pressure as a function of the flow ram pressure. We considered models with different density inhomogeneities in the precursor, parametrized the density contrast $\delta\rho_{rms}/\rho_0$ and by the relative density structures scales λ_ρ/D_{CR} .

In agreement to BJL09, we have found that:

- The interaction of density fluctuations with the CR precursor produces vorticity. Our calculations show that vorticity is generated at a faster rate for higher density contrast and for smaller density structure scales;
- The solenoidal velocity fluctuations dominate over the compressible ones in the turbulence induced at the precursor. Both components achieve roughly $\lesssim 10\%$ of the upstream velocity and their evolution depends strongly on the initial density contrasts and density structures sizes;
- The scales in which the CR pressure induces turbulence in the precursor are close to the scale of the density inho-

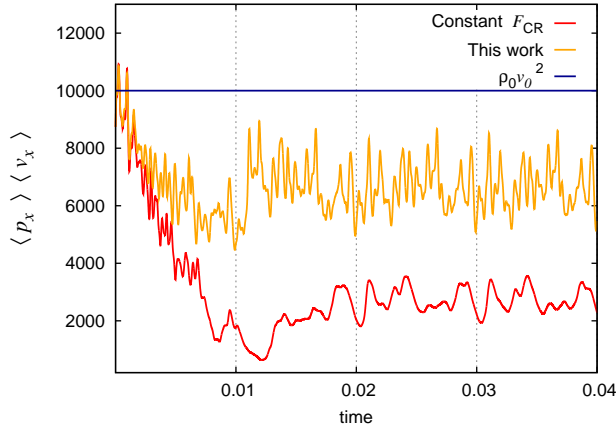


Figure 19. Mean flow evolution of $\langle p_x \rangle \langle v_x \rangle$ at the right boundary, during the whole integration time. Each time period of 0.01 is delimited by a dashed vertical line. Both simulations have synthetic turbulence initial conditions and the same numerical set-up but the force formulation. The constant force corresponds to a force of the form $F_{CR} = -\eta\rho_0 v_0^2/D_{CR}$ (e.g., Drury & Downes 2012; Downes & Drury 2014), and in our set-up F_{CR} is given by Eq. (8). The blue line is the constant ram pressure used in the constant force formulation.

mogeneities. Hence, the injection of turbulent energy in the precursor is larger when the ISM density structures have larger sizes.

The final magnetic energy depends strongly on $\delta\rho_{rms}/\rho_0$, but it is approximately insensitive to the values of λ_p/D_{CR} considered here. In our formulation, the amplification factors of the magnetic field due to the gas compression are relatively small (~ 1.3), being most of the magnetic energy growth due to the small-scale dynamo.

The amplified magnetic fields we obtained in this work are still concentrated at the small scales, indicating that the dynamo do not have enough time to reach the linear phase. The results then depend on the numerical resolution employed, and the magnetic energy amplifications obtained should be considered as lower limits. The maximum amplification factor for the magnetic field we obtained is ~ 14 , for model HAI. When considering a larger domain, the amplification factor duplicates (see Sect. 4.4), therefore the amplified magnetic field that we obtained is $B_{cgs} \geq 100 \mu\text{G}$. This lower limit value is in agreement with the inferred values of the magnetic fields in Cassiopeia A ($\sim 100 \mu\text{G}$) and Tycho's SNRs ($\sim 300 \mu\text{G}$) (Vink & Laming 2003; Cassam-Chenaï et al. 2007).

We found that, under the setup considered, the statistical details of the density fluctuations (power spectrum slope and anisotropies in the density structures) have little influence on the final magnetic field amplification. Therefore, for the same $\delta\rho_{rms}/\rho_0$ and λ_p/D_{CR} , no significant differences were found between density fields built synthetically or obtained from MHD turbulence.

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